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APPROXIMATE ANALYSIS OF LIFTING FORCES ON A WING NEAR A FREE SURFACE

(PRIBLIZHENNYI RASCHET PODEMNOI SILY KRYLA VBLIZI SVOBODNOI POVERKHNOSTI)

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Using the theory of small waves, the problem of the motion of a body submerged in a fluid has been investigated by many authors [1, 2].

With the aid of the general method of N.E. Kochin it is possible to obtain an approximate solution of the problem of the motion of a wing near a free surface.

If we satisfy the conditions of N.E. Zhukovskii's theorem on a small scale for the complex velocity, we can write the expression

$$V(z) = V_\infty(z) + V_2(z) \tag{1}$$

where $V_\infty(z)$ is the complex velocity of the motion of the wing in an unlimited flow,

$$V_2(z) = \frac{1}{2\pi i}\int_C \overline{V_\infty(z)} \left[\frac{1}{z-\bar{\zeta}} - 2i\nu e^{-i\nu z} \int_{+\infty}^{z} \frac{e^{i\nu t}}{t-\bar{\zeta}}\, d\bar{\zeta}\right] \tag{2}$$

For the active forces of the flow we have the expressions

$$P_h = \rho v_0 \Gamma_\infty - \frac{\rho}{2\pi}\int_0^\infty |H(\lambda)|^2 \, d\lambda + \frac{\rho\nu}{\pi}\, \text{v.p.} \int_{-\infty}^{1} |H(\nu - \lambda\nu)|^2 \frac{d\lambda}{\lambda}$$
$$Q = \rho\nu\, |H(\nu)|^2 \tag{3}$$

where Γ_∞ is the circulation around the wing in an unlimited flow and the function $H(\lambda)$ is determined by the expression

$$H(\lambda) = \int_C e^{-i\lambda z} V_\infty(z)\, dz \tag{4}$$

Here and later on, the following designations are used: h is the relative submersion of the wing; b is a chord of the wing taken as a typical dimension; δ is the relative thickness of the wing; ζ is the correction to account for the finiteness of the span of the wing near the free surface; τ is the coefficient representing the shape of the wing viewed from above; α_k is the edge angle; α_0 is the angle of zero lifting force.

Formulas (2), (3) and (4) correspond to the formulas of Kochin in which, instead of $V_h(z)$ and Γ_h at depth h we have $V_\infty(z)$ and Γ_∞. With a Froude number $F = v/\sqrt{gb} \to \infty$ the expression for the lifting force of a flat plate near the free surface is obtained by hypergeometric functions in the form

$$P_h = \rho v_0 \Gamma_\infty - \frac{\rho \Gamma_\infty^2}{4\pi R \sqrt{2}\sqrt{8h^2+1}} F\left(\frac{1}{4}, \frac{3}{4}, 1; \frac{1}{(8h^2+1)^2}\right) - $$
$$- \frac{\rho v_0 \Gamma_\infty \cos\alpha_k}{2}\left[1 - \frac{4h}{\sqrt{2}\sqrt{8h^2+1}} F\left(\frac{1}{4}, \frac{3}{4}, 1; \frac{1}{(8h^2+1)^2}\right)\right] \tag{5}$$

or

$$\gamma_h = \frac{P_h}{P_\infty} = 1 - \frac{\sin\alpha_k}{\sqrt{2}\sqrt{8h^2+1}} F\left(\frac{1}{4}, \frac{3}{4}, 1; \frac{1}{(8h^2+1)^2}\right) - $$
$$- \frac{\cos\alpha_k}{2}\left[1 - \frac{4h}{\sqrt{2}\sqrt{8h^2+1}} F\left(\frac{1}{4}, \frac{3}{4}; 1; \frac{1}{(8h^2+1)^2}\right)\right] \tag{6}$$

For a Zhukovskii airfoil and wing the expressions for γ_h have the form

$$\gamma_h = 1 - \frac{\sin(\alpha_0+\alpha_k)}{\sqrt{2}\sqrt{8h^2+1}\cos\alpha_0} F\left(\frac{1}{4}, \frac{3}{4}, 1; \frac{1}{(8h^2+1)^2}\right) - $$
$$- \frac{\cos\alpha_k}{2\cos 2\alpha_0}\left[1 - \frac{4h}{\sqrt{2}\sqrt{8h^2+1}} F\left(\frac{1}{4}, \frac{3}{4}, 1; \frac{1}{(8h^2+1)^2}\right)\right] \tag{7}$$

$$\gamma_h = 1 - \frac{\sin(\alpha_0+\alpha_k)}{\sqrt{2}\sqrt{8h^2+1}\cos\alpha_0} F\left(\frac{1}{4}, \frac{3}{4}, 1; \frac{1}{(8h^2+1)^2}\right) - \frac{(1+\mu)^2\cos\alpha_k}{2\cos 2\alpha_0} \times$$
$$\times\left[1 - \frac{4h}{\sqrt{2}\sqrt{8h^2+1}} F\left(\frac{1}{4}, \frac{3}{4}, 1; \frac{1}{(8h^2+1)^2}\right)\right] - $$
$$- \frac{k\delta(1+\mu)^4 F\left(^3/_4, {}^5/_4, 2; (8h^2+1)^{-2}\right)}{4\sqrt{2}(8h^2+1)^{3/2}\sin(\alpha_0+\alpha_k)\cos 3\alpha} \tag{8}$$

where k is the ratio of thickness above the chord to the total thickness of the profile

$$\mu = \frac{0.77\,\delta}{1-0.6\,\delta}$$

For the coefficient of the lifting force of a wing of finite span we can write the expression

$$C_{yh} = \frac{\psi\, dC_{y\infty}/d\alpha}{1 + (\psi/\pi\lambda)(dC_{y\infty}/d\alpha)(1+\tau)\zeta}(\alpha_0 + \alpha_k - \Delta\alpha_i) \tag{9}$$

$$\psi = 1 - \frac{2\sin(\alpha_0+\alpha_k)}{\sqrt{2}\sqrt{8h^2+1}\cos\alpha_0} F\left(\frac{1}{4}, \frac{3}{4}, 1; \frac{1}{(8h^2+1)^2}\right) - $$
$$- \frac{(1+\mu)^2\cos\alpha_k}{2\cos 2\alpha_0}\left[1 - \frac{4h}{\sqrt{2}\sqrt{8h^2+1}} F\left(\frac{1}{4}, \frac{3}{4}, 1; \frac{1}{(8h^2+1)^2}\right)\right]$$

$$\Delta\alpha_i = \frac{1}{\psi}\frac{k\delta(1+\mu)^4}{4\sqrt{2}(8h^2+1)^{3/2}\cos 3\alpha_0} F\left(\frac{3}{4}, \frac{5}{4}, 2; \frac{1}{(8h^2+1)^2}\right)$$

The results of calcula"on according to formulas (6) to (9) agree well with the experimental data for all relative submersions [3].

Submitted November 5, 1960

[1.] Trudy konferentsii po teorii volnogo soprotivleniya. [Transactions of a conference on the theory of wave resistance]. TSAGI, 1937.

[2.] Kochin, J.E. Writings. Vol. II, Izd-vo Akad. Nauk SSSR, 1949.

[3.] Chudinov, S.D. "The lifting force of an underwater wing of finite span." Trudy VNITOSS, 1955, Vol. 6.